

High precision QCD at hadron colliders

New techniques and results for perturbative calculations

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Outline

- Motivation and introduction
- Parton distribution functions
 - DGLAP kernels at NNLO
 - PDF errors
- Progress in next-to-leading order calculations
 - Higgs phenomenology
 - New techniques for NLO calculations
 - Merging NLO with parton showers
- Progress in NNLO calculations
 - New techniques for two-loop integrals
 - Understanding infrared divergences at NNLO
 - Phenomenology at NNLO
- Conclusions and outlook

The need for high precision

- Provides strong constraints on the SM and its extensions
 - Incredible success of the LEP, SLC Z -pole program
 - Extraction of M_W , m_t at the Tevatron
 - ⇒ Precision EW data provides a vital experimental handle on new physics models
 - Become consistency checks, discriminators in presence of new physics
- Precise predictions for signals, backgrounds
- Measurements of NP parameters: masses, couplings
- Needed in absence of clear NP effect

Experimental prospects

- Great prospects for precision physics at hadron colliders

- At the **Tevatron**

- Each experiment has $\approx 220 \text{ pb}^{-1}$ physics-ready data
- Expect $2 - 3 \text{ fb}^{-1}$ by LHC turn-on

$$\Delta m_t = \pm 5 \text{ GeV} \rightarrow \pm(2.5 - 3) \text{ GeV}$$

$$\Delta M_W = \pm 60 \text{ MeV} \rightarrow \pm(25 - 30) \text{ MeV}$$

- At the **LHC**

- In 1 year at 10 fb^{-1} : over 10^7 $W, Z, t\bar{t}$ events $\Rightarrow \Delta\sigma_{stat} \ll 1\%$
- Improved systematics (j, l energy scales) from high statistics samples
- Reduction of luminosity error to $1 - 5\%$

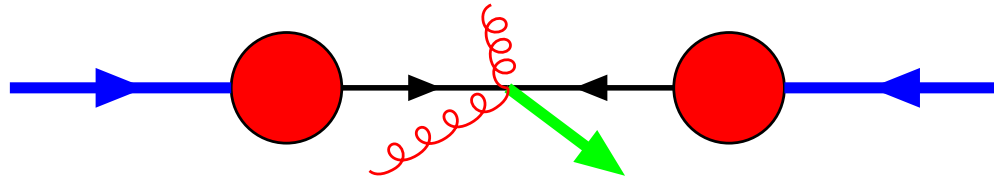
\Rightarrow **Percent level physics at the LHC!**

- Talks by Dissertori, Huston, Wood at KITP conference on Collider Physics

Precision QCD

- Everything at hadron colliders involves QCD!
- Observables in hadronic collisions

$$N_{events} = L \int f_i(x_1, \mu^2) f_j(x_2, \mu^2) \sigma_{ij}(x_1, x_2, \mu^2)$$



- Require
 - luminosity measurement
 - parton distribution functions
 - scattering cross sections

Components of a QCD calculation

● Extract luminosity from well-measured, understood processes

- Total inelastic cross section at the Tevatron

- W, Z cross sections at the LHC

⇒ Will quote $N_X = N_{W,Z} \left(\frac{\sigma_X}{\sigma_{W,Z}} \right)_{th}$

- Theory predictions must account for acceptances

● Extract universal PDFs from experiment

- DIS, jet production, fixed target Drell-Yan

- Theory predictions must allow x dependence of $f(x, \mu^2)$ to be reconstructed

- Evolution of momentum scale μ requires DGLAP kernels:

$$\frac{d f(x, \mu^2)}{d \ln \mu^2} = \sum_{n=0} \left(\frac{\alpha_S}{4\pi} \right)^n P^{(n)}(x) \otimes f(x, \mu^2)$$

Cross sections in QCD

- $\sigma = \sigma_0 \{1 + \alpha_S (l + \sigma_1) + \alpha_S^2 (l^2 + l + \sigma_2) + \mathcal{O}(\alpha_S^3)\}$

$$+ \alpha_S \left\{ \text{diagram 1}, \text{diagram 2} \right\} + \alpha_S^2 \left\{ \text{diagram 3}, \text{diagram 4} \right\}$$

The diagrams represent Feynman diagrams for quark-quark scattering. The first diagram is the tree-level process. The second and third diagrams are one-loop corrections (virtual gluon exchange). The fourth diagram is a two-loop correction (double gluon exchange).

- Strong coupling constant not small: $\alpha_S(M_Z) \approx 0.12$

⇒ higher order corrections important

- Contains scales $l = \ln(\mu^2/Q^2)$

- UV behavior: renormalization scale dependence (μ_R)

- IR behavior: factorization scale dependence (μ_F)

- Scales are arbitrary: $\frac{d\sigma}{d\mu} = 0$

⇒ but truncation of expansion at $\mathcal{O}(\alpha_S^n)$ induces a scale dependence of $\mathcal{O}(\alpha_S^{n+1})$

- Residual scale dependences provide estimate of neglected higher order effects

From LO to NNLO

• LO

- No quantitative estimate of cross section
- Few partonic channels open in initial state
- ⇒ poor estimate of kinematics, dependence on PDFs
- Few partons in final state ⇒ poor modeling of jets

• NLO

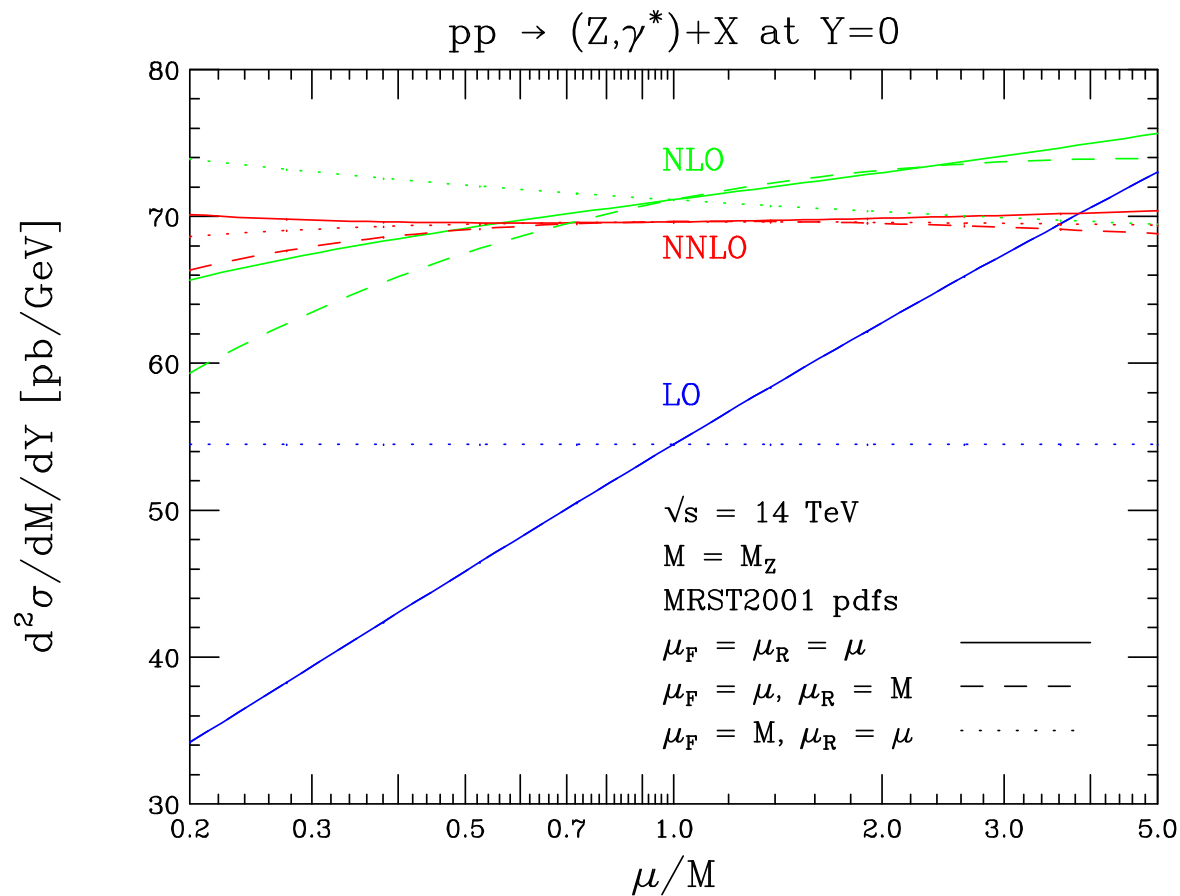
- First quantitative estimate of cross section
- Better modeling of kinematics, final-state structure

• NNLO

- Residual scale dependences small
- Allows precision predictions

High precision theory

- In the best of all possible worlds:



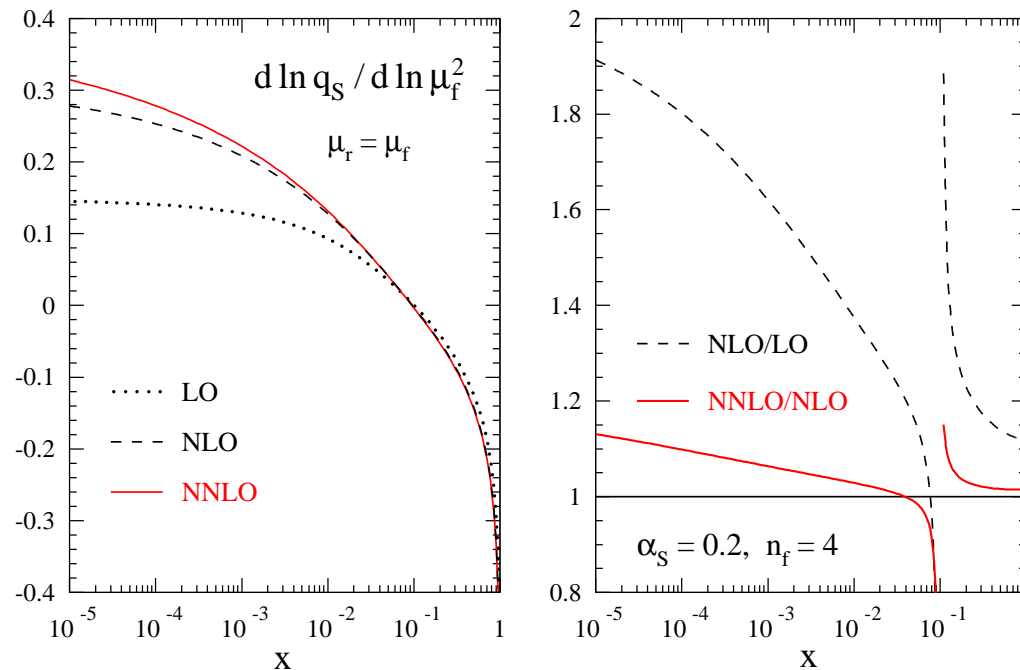
Parton distribution functions

Parton distribution functions

- Method of extraction
 - Choose a data set for a given process (DIS, Drell-Yan, jets)
 - Write theory prediction as a convolution of PDFs and hard scattering cross section, at a given order
 - Extract PDFs, to the given order
 - Evolve to other Q^2 with the DGLAP equation
- Several different sets available: CTEQ, MRST, Alekhin, ...
 - "NNLO" PDFs provided by MRST, Alekhin
- Sources of error
 - In the evolution: DGLAP kernels not known to needed order in α_S
 - In the fitting: experimental errors, imprecise hard scattering cross sections, ...

DGLAP evolution

- Full calculation of **NNLO** kernels recently completed
(Moch, Vermaseren, Vogt)



- Corrections $5 - 10\%$ for $x < 10^{-3}$
- New $\ln^4 x$ LL structure
- μ variation $1 - 2\%$ for $x > 10^{-3}$
 $< 8\%$ for $x < 10^{-3}$
- **N³LO** likely important for small x

- Agrees with approximate result based on first few moments
 - "NNLO" PDFs of MRST, Alekhin likely okay for most phenomenological purposes

PDF errors

● Recent efforts to estimate PDF errors on W, Z, H production

- Variations within a PDF set are small

⇒ $\delta\sigma_W^{NNLO} = \pm 4\%$, $\delta\sigma_H^{NNLO} = \pm 3\%$ for MRST, similar for Alekhin

- Variations between sets larger

⇒ $\approx 15\%$ for H production (Djouadi, Ferrag)
(at NLO, but quoted variations within set were $\approx 5 - 10\%$)

⇒ $\leq 8\%$ for W, Z production at the LHC, at NNLO

● In the future (?)

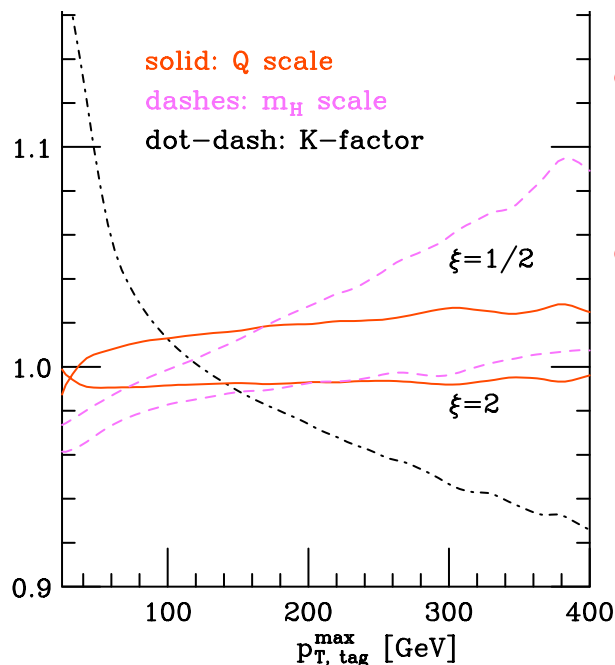
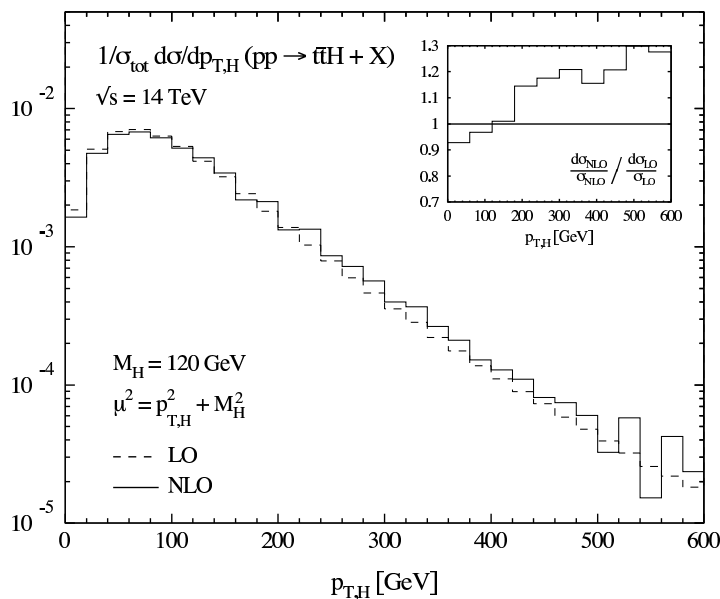
- Use full NNLO kinematics for Drell-Yan cross section in fit (only inclusive K-factor)
- Jet cross sections at NNLO
- Add Drell-Yan to Alekhin (only DIS, others global)
- Use LHC data to constrain

Progress in NLO calculations

Advances in NLO Phenomenology

• Several studies of Higgs physics at the LHC

- $pp \rightarrow t\bar{t}H, b\bar{b}H$: Beenakker *et. al.*; Dawson *et. al.*
- $pp \rightarrow jjH$ (WBF): Figy, Oleari, Zeppenfeld; Berger, Campbell



• K-factors vary with kinematics

• Uncertainties:

WBF: $\pm 6 - 10\%$

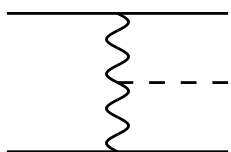
$t\bar{t}H$: $\pm 15 - 20\%$

• No longer just discovery; detailed analysis of couplings, etc.

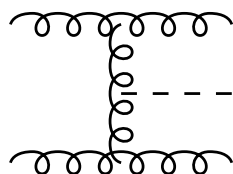
Extracting Higgs couplings

- Measure HWW coupling with WBF (ATLAS; Berger, Campbell)

Signal: WBF

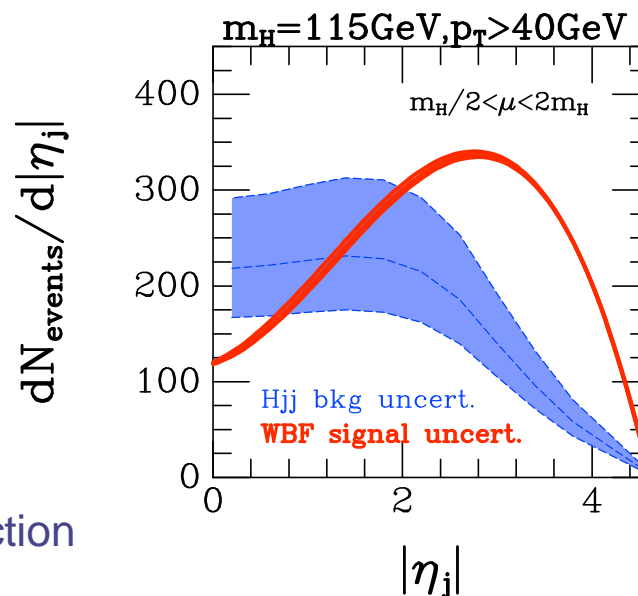


Background: QCD Hjj



⇒ Higgs production now a background!

- Separate S, B with kinematics
- Uncertainty dominated by $\delta S/S, \delta B/B$
 $\delta B/B = \pm 20\%, \delta S/S = \pm 4\%$ (ATLAS)
 $\delta B/B = \pm 30\%, \delta S/S = \pm 10\%$ (BC)
- Estimate $\delta g/g \approx 10\%$ after 200 fb^{-1} (BC)
- Background known only at LO
 ⇒ need NLO computation of QCD Hjj production



Wishful thinking

- Missing many needed **NLO** computations

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An experimenter's wishlist

- Hadron collider cross-sections one would like to know at NLO

Run II Monte Carlo Workshop, April 2001

Single boson	Diboson	Triboson	Heavy flavour
$W + \leq 5j$	$WW + \leq 5j$	$WWW + \leq 3j$	$t\bar{t} + \leq 3j$
$W + b\bar{b} + \leq 3j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \leq 3j$	$WW + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 5j$	$ZZ + \leq 5j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 3j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 2j$
$Z + c\bar{c} + \leq 3j$	$ZZ + c\bar{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 2j$
$\gamma + \leq 5j$	$\gamma\gamma + \leq 5j$		$b\bar{b} + \leq 3j$
$\gamma + b\bar{b} + \leq 3j$	$\gamma\gamma + b\bar{b} + \leq 3j$		
$\gamma + c\bar{c} + \leq 3j$	$\gamma\gamma + c\bar{c} + \leq 3j$		
	$WZ + \leq 5j$		
	$WZ + b\bar{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 3j$		
	$Z\gamma + \leq 3j$		

Wishful thinking

- Missing many needed **NLO** computations

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Theoretical status

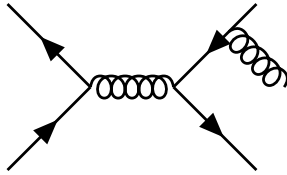
- Much smaller jet multiplicities, some categories untouched

Single boson	Diboson	Triboson	Heavy flavour
$W + \leq 2j$	$WW + \leq 0j$	$WWW + \leq 3j$	$t\bar{t} + \leq 0j$
$W + b\bar{b} + \leq 0j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \leq 0j$	$WW + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 2j$	$ZZ + \leq 0j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 0j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 0j$
$Z + c\bar{c} + \leq 0j$	$ZZ + c\bar{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 0j$
$\gamma + \leq 1j$	$\gamma\gamma + \leq 1j$		$b\bar{b} + \leq 0j$
$\gamma + b\bar{b} + \leq 3j$	$\gamma\gamma + b\bar{b} + \leq 3j$		
$\gamma + c\bar{c} + \leq 3j$	$\gamma\gamma + c\bar{c} + \leq 3j$		
	$WZ + \leq 0j$		
	$WZ + b\bar{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 0j$		
	$Z\gamma + \leq 0j$		

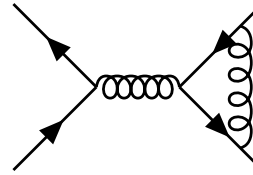
Computing cross sections at NLO

- Two components of an NLO calculation:

Real:



Virtual:



- Obtain a cross section in the form:

$$\sigma_{NLO} = \int d\Phi_n (\sigma_B + \alpha_S \sigma_{virt}) + \alpha_S \int d\Phi_{n+1} \sigma_{real}$$

- Dealing with divergences

- UV: cancel with coupling constant renormalization
- IR: typically use **dipole subtraction** (Catani, Seymour)
 - Introduce counterterm D which reproduces IR divergences of σ_{real} :

$$\sigma_{NLO} = \int d\Phi_n (\sigma_B + \alpha_S [\sigma_{virt} + D_I]) + \alpha_S \int d\Phi_{n+1} [\sigma_{real} - D] ,$$

$$\text{with } D_I = \int d\Phi_1 D$$

- Cancel divergences analytically in $\sigma_{virt} + D_I = \sigma_{virt}^{fin}$
- $\sigma_{real} - D$ is pointwise finite, numerically integrable

Obstacles at NLO

- Two major sticking points at NLO:
 - Going beyond $2 \rightarrow 3$ processes
 - Large number of processes needed
- Root cause: multi-leg ($N \geq 5$) virtual integrations
 - Many scales ($s_{ij}, M_W, m_H, m_t, \dots$)
 - \Rightarrow expressions become enormous
 - \Rightarrow large numerical cancellations between terms
 - \Rightarrow different integrals needed for each process
 - Many singular regions: soft, collinear, UV, thresholds, spurious singularities, ...
 - \Rightarrow singular subtractions not as well understood as for real emission contributions
- Goal: automated, general method, as for real contributions
 - Also want a flexible approach to be ready for LHC analyses

Schematic of NLO virtual corrections

- What we get from Feynman diagrams for $2 \rightarrow N - 2$:

$$I_N^m = \int d^d k \frac{k^{\mu_1} \dots k^{\mu_m}}{[(k + q_1)^2 - m_1^2] \dots [(k + q_N)^2 - m_N^2]}$$

- Large number of integrals which aren't independent
- ⇒ can reduce tensor structure, use recurrence relations to obtain a minimal set of basis scalar integrals

$$\text{Hexagon}(\{k^{\mu_i}\}) = \sum c_T^i \text{Triangle} + \sum c_B^i \text{Square}$$

- c^i lengthy for large N , many scales
- $c^i \sim 1/p_a \cdot p_b$
- Expressions for basis integrals complicated

- How much do we do **analytically**, how much **numerically**?

Automating NLO virtual corrections

● Hybrid approach (Giele, Glover)

- Reduce divergences to triangle integrals
- Solve the remaining recurrence relations numerically
- Completely avoid lengthy ϵ^i

● Numerical approach (Nagy, Soper)

- Define counterterms for UV, IR, collinear singularities graph-by-graph
- Similar to dipole subtraction for real contributions
- Integrate counterterms analytically, feed remainder directly to numerical integration

● Other approaches suggested (Binoth *et. al.*, ...)

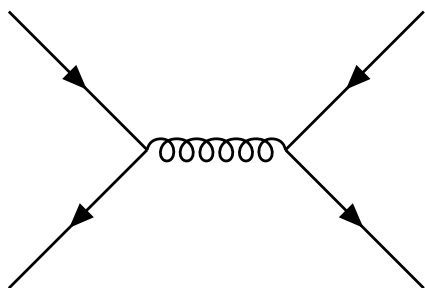
⇒ No implementation yet of any method

Merging parton showers with NLO

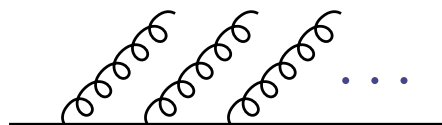
Merging parton showers with NLO

- Experimentalists typically use shower MCs for predictions

Begin with:



MC generates shower for each line:



- Emissions controlled by Sudakov form-factor:

$$\Delta(x, x_M) = \exp \left(-\alpha_S \int_x^{x_M} d\Phi Q(\Phi) \right)$$

- $Q(\Phi)$ encodes behavior of the soft/collinear emissions
- Typically use several approximations: no shower interference, angular ordering, ...

Fixed order vs. shower MCs

● Fixed order

- + Systematic expansion in α_S
- + Based on exact matrix elements; describes hard/wide angle emissions well
- Relatively few partons in final state; no way to hadronize
- Not available beyond leading order for all processes; when available, tend to be spread among different codes

● Shower Monte Carlos

- + Generate many partons in the final state; access to hadronization
- + Many processes available in a few codes (HERWIG, PYTHIA)
- Doesn't describe hard/wide angle emissions correctly
- Doesn't systematically include higher order corrections \Rightarrow can't do precision physics

\Rightarrow Want the advantages of both approaches

Merging parton showers with NLO

- Can't just use NLO matrix elements in the MC

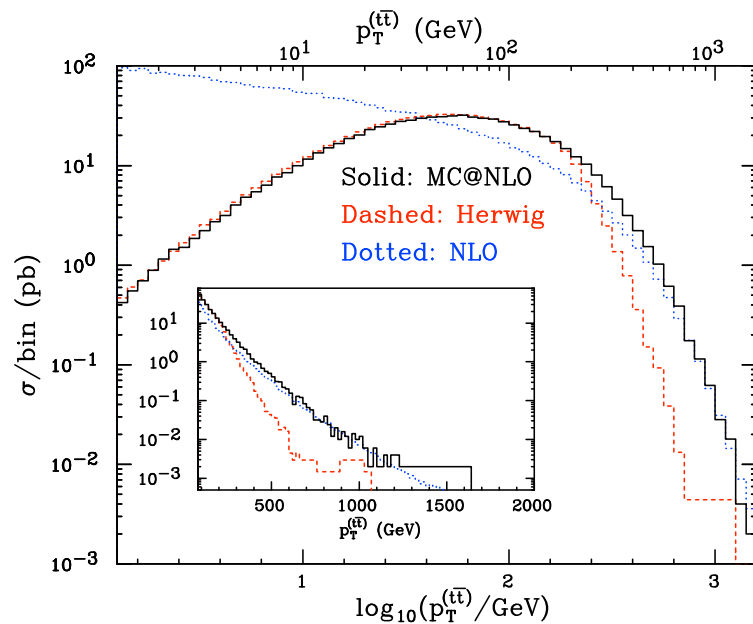
- $\sigma_{NLO} = \int d\Phi_n (\sigma_B + \alpha_S [\sigma_{virt} + D_I]) + \alpha_S \int d\Phi_{n+1} [\sigma_{real} - D]$

$$\sigma_{real} = \left| \begin{array}{c} \text{diagram 1} \\ \text{diagram 2} \\ + \dots \end{array} \right|^2 \Rightarrow \text{NLO corrections already include some gluon emissions; double-counting}$$

The diagram shows two incoming lines (arrows) and two outgoing lines (arrows). In the first diagram, a gluon (curly line) is emitted from the top-left line. In the second diagram, a gluon is emitted from the bottom-left line. The diagrams are enclosed in large vertical bars, and a red superscript '2' is placed to the right of the bars. Ellipses follow the diagrams, indicating more terms in the sum.

- Incompatible with subtraction formalism for NLO corrections
 - D is the soft/collinear limit of $\sigma_{real} \Rightarrow$ has only n -body kinematics
 - Generate different showers for D, σ_{real}
- \Rightarrow only cancel divergences after generating showers for each piece

- Use the MC itself as a counterterm (Frixione, Webber)
 - $Q(\Phi)$ encodes emission singularities \Rightarrow use it as an additional counterterm
 - $Q(\Phi)$, σ_{real} coincide in singular phase space regions, so weights are finite
 - Also removes double counting of real emissions



- Smoothly matches **soft/collinear** (MC) and **hard** (NLO) regions
- Works for **most** observables; MC not a local counterterm for large-angle soft emissions

Progress in NNLO calculations

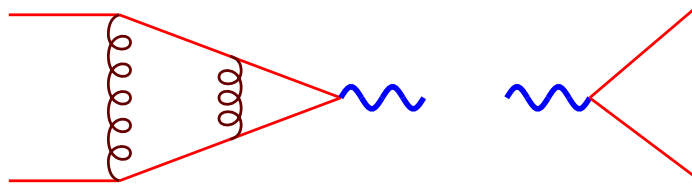
The NNLO revolution

- Tremendous progress recently in NNLO computations
 - New computational techniques for two-loop integrals
 - Better understanding of singular structure of real radiation
 - Many new phenomenological results
- Is NNLO necessary?
 - Reduced scale dependence
 - More partons \Rightarrow more realistic
 - Several concrete physical applications that require NNLO:
 - Higgs production at hadron colliders
 - Drell-Yan (luminosity monitor, PDF measurements)
 - Jet production at hadron colliders (PDFs, α_S extraction)
 - Jet production at e^+e^- colliders

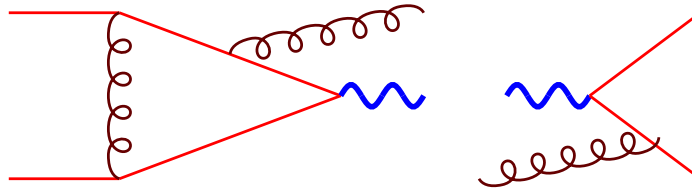
$$\alpha_S(M_Z) = 0.1202 \pm 0.0003(\text{stat}) \pm 0.0009(\text{sys}) \pm 0.0009(\text{had}) \pm 0.0047(\text{th})$$

Anatomy of a NNLO calculation

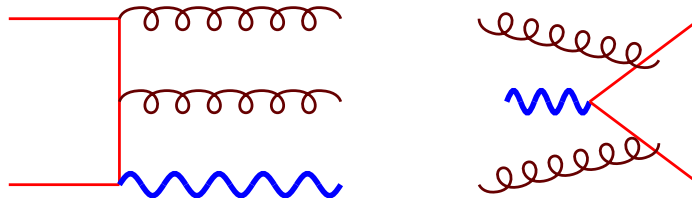
- Virtual-Virtual



- Real-Virtual



- Real-Real

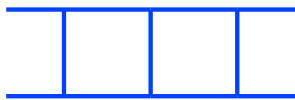


Two-loop integrals

- Loop integrals satisfy recurrence relations arising from Poincare invariance
 - Use **integration-by-parts** to derive (Chetyrkin, Tkachov)
 - $I[\nu_1, \nu_2] = \int d^D k \frac{1}{[k^2]^{\nu_1} [(k+p)^2]^{\nu_2}}$
 - Set $\int d^D k \frac{\partial}{\partial k_\mu} \frac{k^\mu}{k^2 (k+p)^2} = 0$
 - Derive $I[1, 2] = -\frac{D-3}{p^2} I[1, 1]$
- Reduce to a small set of **master integrals**
- Two things to do:
 - Reduce the integrals appearing in the matrix elements to master integrals
 - Calculate the master integrals

Recent virtual progress

- Until recently, missing the master integrals



⇒ calculated by Smirnov, Tausk

- New methods for solving systems of recurrence relations

- **Old-fashioned method:** manipulate recurrence relations manually

⇒ avoids introducing unneeded integrals, but rapidly becomes difficult

- **Algorithmic method** (Laporta):

- Fully automated method of iteratively solving recurrence relations
- Very general procedure applicable to a large class of problems
- Efficient implementation now publicly available (Anastasiou, Lazopoulos)

Available two-loop amplitudes

● Recently computed amplitudes for $2 \rightarrow 2$ processes:

- Two-loop Bhabha scattering in massless QED Bern, Dixon and Ghinculov
- All two-loop $2 \rightarrow 2$ QCD processes. Anastasiou, Glover, Oleari and Tejeda-Yeomans
Bern, De Freitas, and Dixon
- $\gamma\gamma \rightarrow \gamma\gamma$ Bern, Dixon, De Freitas, A. Ghinculov and H.L. Wong
- $gg \rightarrow \gamma\gamma$. (Background to Higgs decay.) Bern, De Freitas, Dixon
- $\bar{q}q \rightarrow \gamma\gamma$, $\bar{q}q \rightarrow g\gamma$, $e^+e^- \rightarrow \gamma\gamma$ Anastasiou, Glover and Tejeda-Yeomans
- $e^+e^- \rightarrow 3$ partons Garland, Gehrmann, Glover, Koukoutsakis and Remiddi
Moch, Uwer, Weinzierl
- DIS 2 jet and $pp \rightarrow W, Z + 1$ jet Gehrmann and Remiddi

Bern

Recent real progress

- Currently the sticking point in completing NNLO calculations
 - Until recently, no direct calculation of $e^+e^- \rightarrow 2 \text{ jets}$ at NNLO!
 - Tree graphs, so what's the problem?
 - ⇒ Understanding their singular structure when partons become unresolved
 - How do we extract their IR singularities before integrating over phase space?
- Two ways to approach the problem:
 - (1) General method which aims for a complete understanding of IR structure
 - (2) Ask for information about restricted, "semi-inclusive" quantities

IR structure at NNLO

● Understanding IR singularities at NNLO

- Would allow for completely differential NNLO calculations
- Extensions of the **subtraction method** to NNLO (Campbell, Glover; Kosower; Weinzierl; Gehrmann-De Ridder, Gehrmann, Glover; Kilgore)

$$\begin{aligned}\sigma_{NNLO} = & \int d\Phi_n \left(\sigma^{(0)} + \alpha_S \left[\sigma_v^{(1)} + D^{(1)} \right] + \alpha_S^2 \left[\sigma_v^{(2)} + D^{(2)} \right] \right) \\ & + \alpha_S \int d\Phi_{n+1} \left[\sigma_r^{(1)} - D^{(1)} \right] + \alpha_S^2 \int d\Phi_{n+2} \left[\sigma_r^{(2)} - D^{(2)} \right]\end{aligned}$$

- Integrate the $D^{(1,2)}$ analytically, and the remainder numerically
- $D^{(2)}$ must incorporate many limits: 3 collinear, 2 pairs collinear, 1 soft + 2 collinear, ...
- **Alternative approach:** Φ_n structure permits an automated extraction of IR divergences (Binoth, Heinrich; Anastasiou, Melnikov, FP)
 - Derive a series in $1/\epsilon$ with **numerical** coefficients
 - Don't need any analytic integrations
 - Don't need to consider singular limits separately

Semi-inclusive observables at NNLO

- Can adapt multi-loop techniques to phase space integrals
(Anastasiou, Melnikov)

$$\sigma_{\alpha\beta\rightarrow 1\dots n} \propto \int \left[\prod_{i=1}^n d^d q_i \delta(q_i^2 - m_i^2) \right] \delta(p_{\alpha\beta} - q_{1\dots n}) |\mathcal{M}_{\alpha\beta\rightarrow 1\dots n}|^2$$

- Cutkosky rules: $\delta(q_i^2 - m_i^2) \Rightarrow \frac{1}{q_i^2 - m_i^2 - i\epsilon} - \frac{1}{q_i^2 - m_i^2 + i\epsilon}$
- Maps phase space integrals \Rightarrow cut loop integrals
- Can extend to differential quantities (Anastasiou, Dixon, Melnikov, FP)
- Rapidity distributions ($u = \frac{x_1}{x_2} e^{-2Y}$):

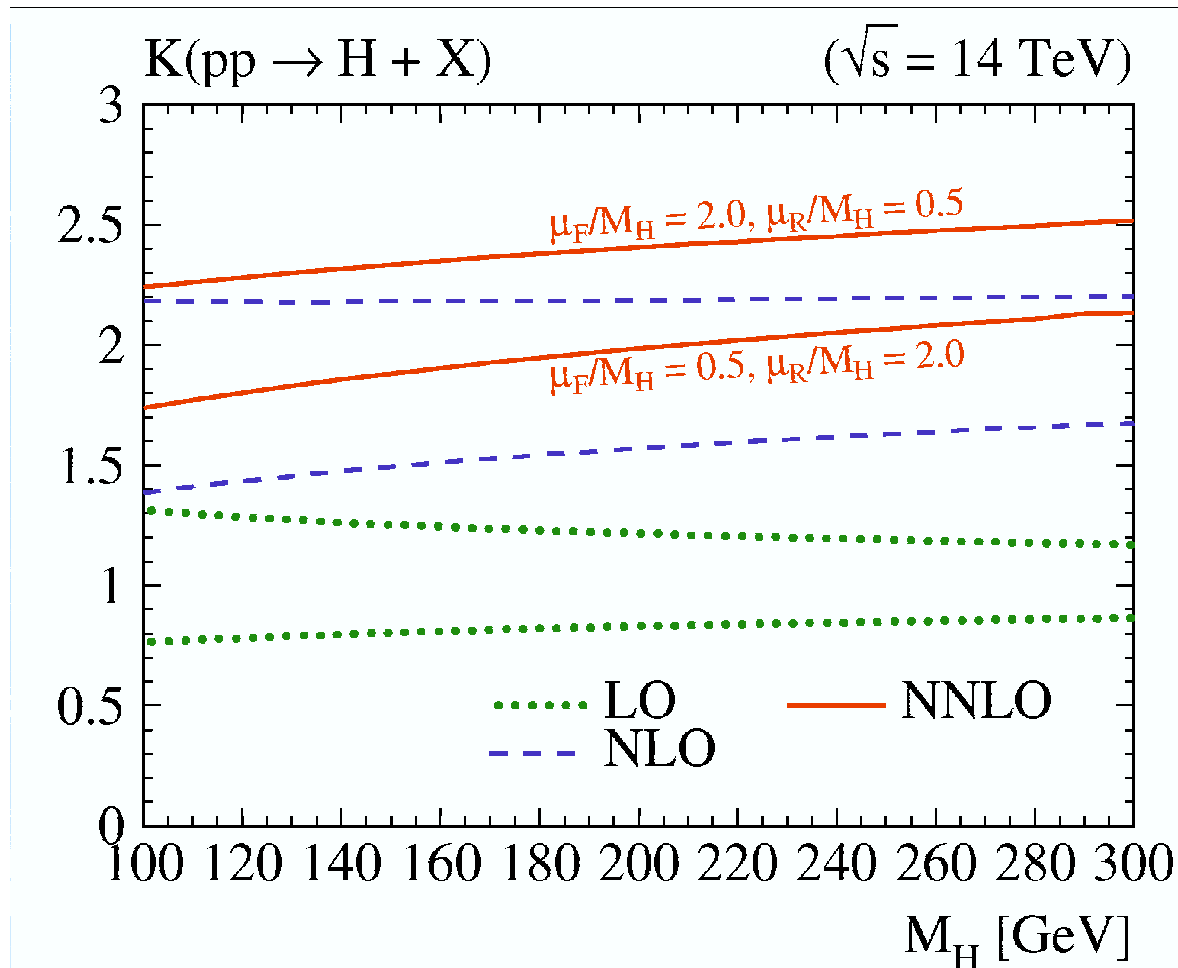
$$\frac{d\sigma}{dY} \propto u \int \left[\prod_{i=1}^n d^d q_i \delta(q_i^2 - m_i^2) \right] \delta\left(u - \frac{p_1 \cdot P_h}{p_2 \cdot P_h}\right) \delta(p_{\alpha\beta} - q_{1\dots n}) |\mathcal{M}_{\alpha\beta\rightarrow 1\dots n}|^2$$

- Make the same replacement for the rapidity constraint
- \Rightarrow Introduce a fictitious particle, whose mass-shell condition \Leftrightarrow phase-space constraint
- In the fully differential limit, recurrence relations provide no information

Higgs production at NNLO

- Several recent **NNLO** calculations

(Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven)

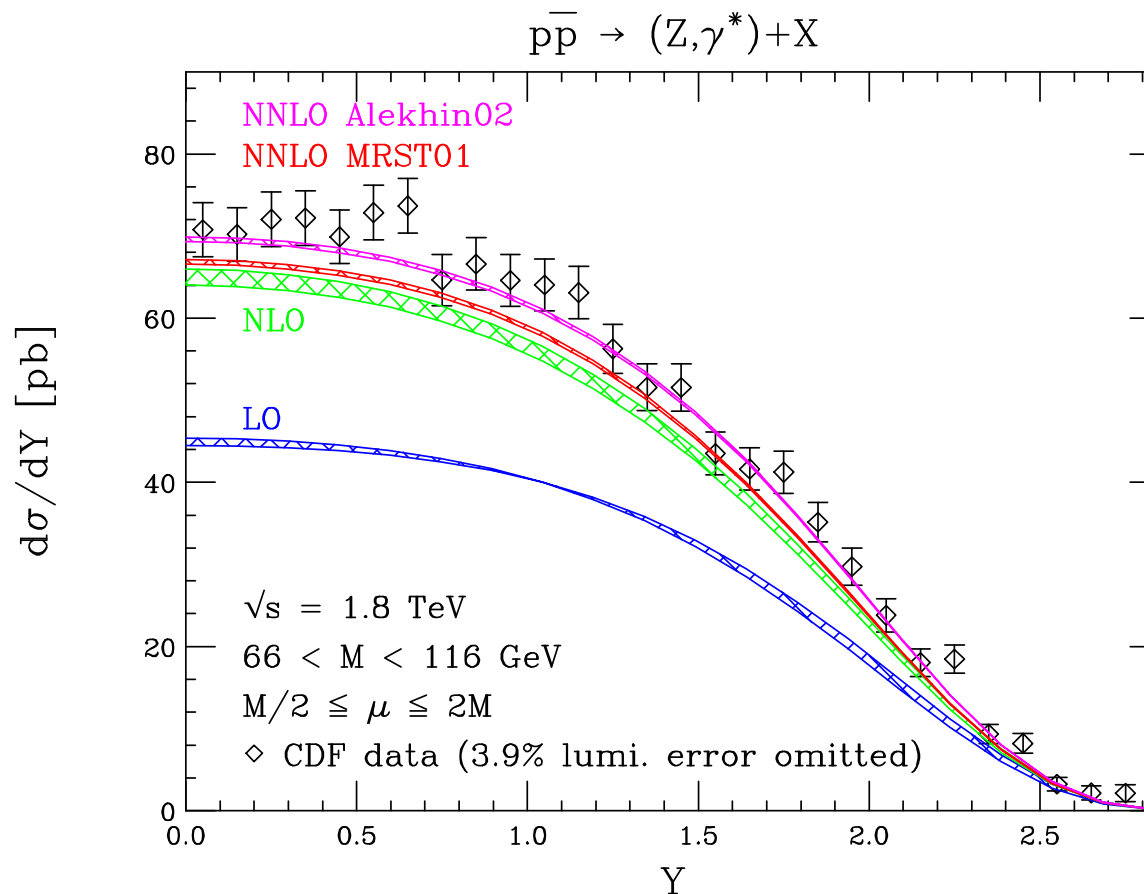


- 30 – 40% residual scale dependence at NLO
- NLO corrections increase LO result by 70 – 80%
- \Rightarrow Does the series converge?
- 20% residual scale dependence at NNLO
- NNLO corrections are $\leq 30\%$

Drell-Yan rapidity distributions

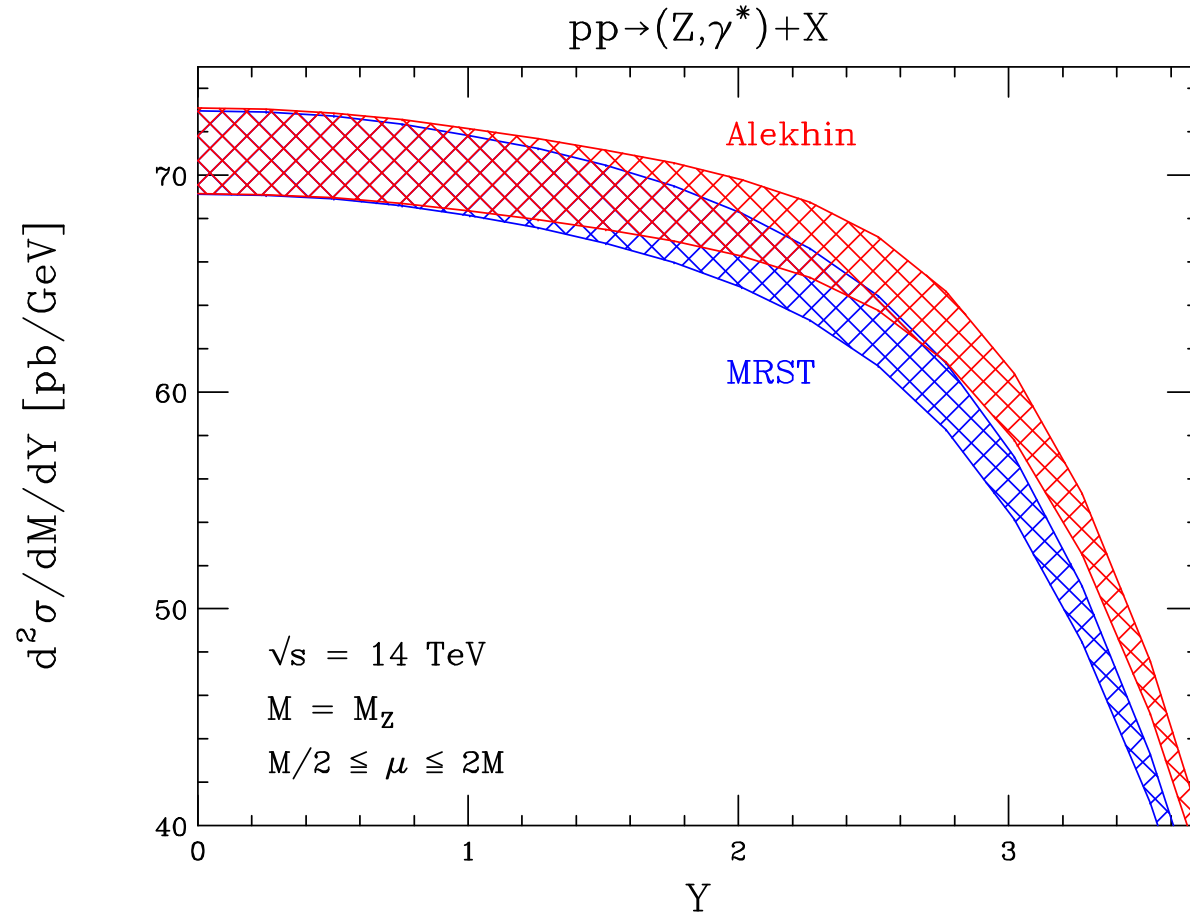
• First complete differential result at NNLO

(Anastasiou, Dixon, Melnikov, FP)



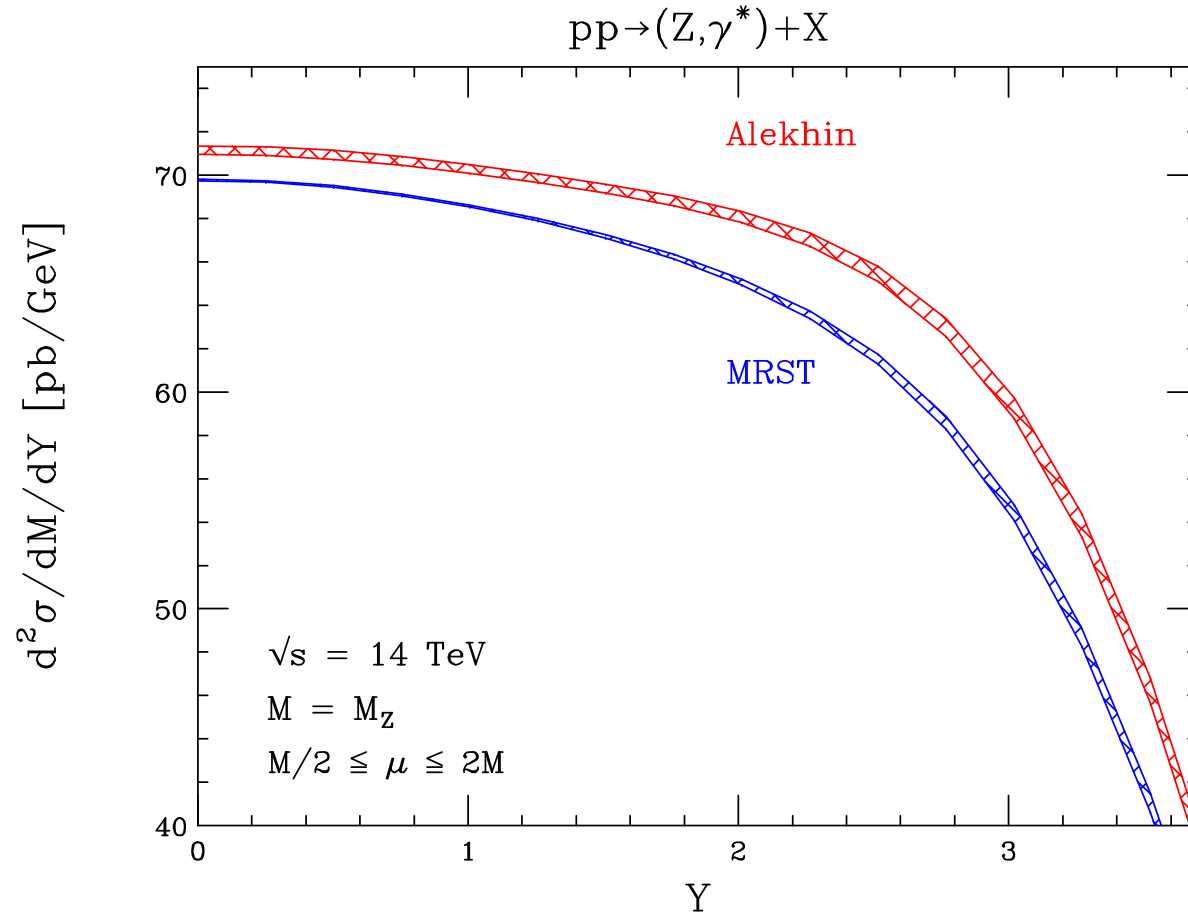
- NNLO corrections increase NLO result by **3-5%**
- Scale variations **3-6%** at NLO, **< 1%** at NNLO
- Drell-Yan now a high precision probe of QCD

PDFs with NLO Drell-Yan



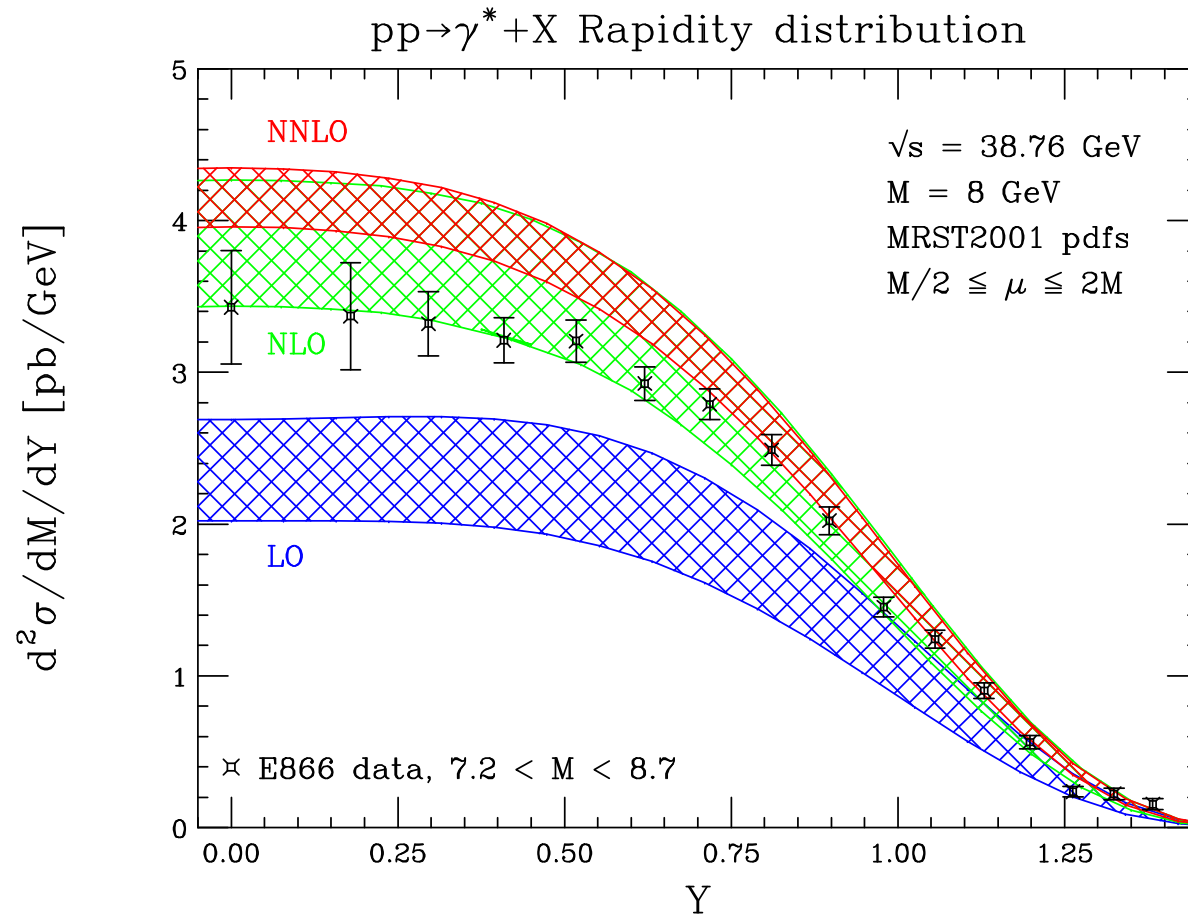
- Alekhin parameterization fits only to DIS data; MRST fits to DIS, DY, jets
- Scale variations render undistinguishable at NLO

PDFs with NNLO Drell-Yan



- Alekhin parameterization fits only to DIS data; MRST fits to DIS, DY, jets
- Scale variations render undistinguishable at NLO
- Resolved at NNLO

Fixed target (E866)



- Strong constraint on \bar{q} and $x \rightarrow 1$ q_{val} distribution functions
 - Reduced μ dependence at NNLO reveals discrepancy with data
- \Rightarrow Tune \bar{q} PDFs

Conclusions

- Exciting prospects for precision physics at future colliders
 - Need theoretical work to fully utilize results
 - Much more to do before LHC start
 - Expect continued progress on several fronts
 - Practical implementations of algorithms for **NLO** calculations
 - Further development of **NNLO** subtraction scheme
 - First completely differential **NNLO** calculations for high-value observables (W, Z, H, \dots)
 - Not yet just turning the crank
- ⇒ room for new ideas!